A Multi-Stage Approach to Air-Rail Competition: Focus on Rail Agency Objective, Train Technology and Station Access

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Abstract

This paper proposes a three-stage game-theoretic model to study air-rail competition in intercity passenger corridors. We consider duopolistic markets, in which a rail agency competes against an airline to maximize their respective objectives. The objective of the rail agency can be maximizing profit or social welfare, or a mix of the two. At the top stage, the rail agency first determines the train technology, which presents its most strategic decision. Then the rail agency and the airline compete sequentially on frequency and fare. We find that, from the operational standpoint, high speed rail (HSR) is always the best technology choice, conditional on availability of high speed infrastructure. This is invariant to the objective of the rail agency. The rail agency focusing more on social welfare will benefit passengers through reduced fare and greater frequency. Rail will also gain more demand and a higher market share. Improving rail stations access is found to have significant impact on rail market share. The impact of rail access time on the rail agency’s objective diminishes with corridor distance.

Keywords: Intercity passenger rail; game-theoretic model; rail agency objective; airline; train technology; rail station access

1 Introduction

Rail transport is gaining growing importance in the US intercity transportation system. Since 1997, Amtrak, the primary US passenger rail carrier, has experienced 55% ridership increase, faster than aviation and other major transportation modes (Brookings, 2013). The flourishing of intercity passenger rail in the US is accompanied by the development of High Speed Rail (HSR) and Higher Speed Rail (HrSR) lines, to which the Congress assigned eight billion dollars for construction as a part of the American Recovery and Reinvestment Act in 2009. However, this unprecedented rail investment is still in great shortage compared to the amount requested by states and Amtrak to develop faster train services. To date, 39 states, the District of Columbia, and Amtrak have submitted applications to the Federal Railroad Administration requesting more than $75 billion for HSR and HrSR projects and corridors in every region of the country. It is expected that, in the foreseeable future, the coexistence of three train technologies, i.e., conventional Amtrak, HrSR, and HSR, will feature the US intercity passenger rail system.

Compared to conventional Amtrak trains, most of which can go no faster than 79 mph as stipulated by federal regulations (US Government Printing Office, 2014) due to the lack of automatic cab signal, train stop, or train control system onboard, HSR and HrSR technologies can allow trains to operate at much higher speeds. A universal definition of HSR and HrSR does not exist, however. According to Peterman et al. (2012), a major distinction between HSR and HrSR is that HSR uses dedicated, electrified rail lines, with speed over 150 mph and up to 220 mph; whereas HrSR trains often run on shared tracks with freight trains, at a lower speed up to 150 mph. A prominent example of HrSR in the US is the Chicago-St. Louis line on which diesel powered passenger trains are designed to operate at a top speed of 110 mph. Besides speed, cost differences also exist for trains with different technologies. It is important to understand the impact on intercity travel of rail services using those technologies.

Train technology determines train operating speed, which further relates to line-haul trip time of rail travelers. In the US, rail faces major competition from commercial aviation. Although the operating speed of

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rail is lower than air transport even with the HSR technology, rail can gain its advantage in total travel time by providing convenient access to train stations. Train stations in the US are mostly located in downtown areas. In contrast, many airports are far away from city centers, resulting in additional travel time to the departure airport, checking in, going through security, and leaving the arrival airport for the final destination. On the other hand, unlike European or Asian cities, US cities are generally more dispersed, which suggests longer average access to rail stations. In intercity passenger rail planning, especially HSR/HrSR line development, choice of train technologies and convenience of rail station access can critically affect the competitiveness of rail in the intercity travel market.

Successful intercity passenger rail service may also depend on the ownership nature of the rail agency. Since its creation in 1970, Amtrak has been operated and managed as a for-profit federal government corporation, which suggests Amtrak’s mixed objective of maximizing profit and social welfare. Although the exact ownership and operational forms of future HSR and HrSR lines in the US are yet to be determined, HSR operators in many other parts of the world are often not just profit seeking but care about social welfare, especially given the considerable public investment involved (Yang and Zhang, 2012). Yet, how private and public objectives affect the attractiveness of rail, especially its competitiveness against air transport services, remains a topic that is not thoroughly examined.

This paper contributes to the literature by developing a three-stage game-theoretic model to characterize air-rail competition in intercity transportation. The research offers new insights into the roles played by the objectives of the rail agency, train technology choice, and rail station access. We focus on intercity corridors with one rail agency and one airline. While this situation is very common for rail, the presence of a single airline best represents corridors that connect between a city which is an airline’s hub and a spoke city. Examples of such corridors in the US are Denver, Colorado (United Airline’s hub) to Albuquerque, New Mexico; Minneapolis, Minnesota (Delta Airline’s hub) to Madison, Wisconsin; and Phoenix, Arizona (US Airways’ hub) to Las Vegas, Nevada. These corridors are all included in the US National Rail Plan (FRA, 2009). At the top stage of the game, we investigate the best choice of train technology (conventional/Amtrak, HrSR, HSR) characterized by different operating speeds. For aircraft, there is very limited room to change speed. Then, best service frequency and fare levels are determined for each mode at the middle and bottom stages. The decisions on train speed, service frequency, and fare are investigated under a range of scenarios that vary by travel distance, the objective of the rail agency, and train station access.

The remainder of the paper is organized as follows. In Section 2, a review of existing literature on air-rail competition is conducted. We synthesize the major approaches and identify the research gaps in previous studies, based on which our contributions are highlighted. Section 3 presents the three-stage game-theoretic model. Section 4 is dedicated to numerical analysis. There we analyze equilibrium fare, service frequency, and train technology choices under a range of scenarios with different market distances, rail operating objectives, and rail station access. Summary of key findings and their interpretations are offered in Section 5.

2 Literature review and research contributions

Existing studies addressing air-rail competition can be largely grouped into two categories: 1) random utility based models which intend to capture mode choice behavior of travelers; and 2) game-theoretic models that characterize the dynamics between travelers and transportation service providers, as well as interactions among different transportation service providers.

Among many studies in the first group, Yao and Morikawa (2005) develop an integrated intercity travel demand modeling system to investigate the effect on mode share of changes in transportation service levels. An accessibility measure is defined to capture short-term induced travel demand. Using data collected from a stated preference survey, Park and Ha (2006) evaluate the impacts on domestic aviation demand of the first HSR service in Korea and conclude that travelers are more likely to choose HSR than air in domestic travel. The authors find much greater fare elasticity than frequency elasticity. Fu et al. (2014) use aggregate OD market data to estimate the impact on air-rail competition in the Tokyo-Osaka corridor by introducing a super HSR service. A Nested Logit model taking into account the “outside option” is estimated. However, this could be a potential weakness, as doing so may not be able to explicitly capture the introduction of new modes. The authors find that Japanese consumers do not consider air and rail services as close substitutes, and they are more sensitive to travel time and frequency than ticket price. The new HSR service will force airlines out of the market while inducing substantial new rail passenger demand. A different view is obtained by Roman et al. (2007) on the competitiveness of air and HSR for the Madrid-Barcelona market. Even with degraded air service qualities, such as increasing air transport waiting, access, and flight delay, the market share of HSR would still
be under 35%. Different from the previous studies that consider service frequency as one service attribute, Cascetta and Coppola (2012) develop a schedule-based model for intercity travel in Italy, which assigns travelers to individual trains, thus allowing for forecast of seat occupancy on an individual train basis for each station-to-station link. Using extensive traffic counts and stated and revealed preference interviews, the authors calibrate the model and analyze the correlation structure of a nested logit model, which captures the substitution pattern among alternatives. They find that the correlation structure significantly varies by the trip purpose.

Turning to the game-theoretic approach, Adler et al. (2010) develop a network-based game engineering tool to evaluate competition among HSR, legacy carriers, and low-cost carriers in medium- to long-haul markets in the entire Trans-European Network (TEN). Under the assumption that HSR and airlines maximize their respective profits, the authors conclude that some of the Trans-European HSR projects are justified despite their substantial fixed costs. Yang and Zhang (2012) use an adapted Hotelling (differentiated Bertrand) model for air-HSR competition, in which the HSR operator is assumed to maximize a weighted sum of social welfare and profit. The authors extend the model to consider price discrimination by the airline and numerically solve six equilibria with simultaneous decisions on fare and frequency. The authors find that price discrimination increases airline profit but keeps HSR profit unchanged. For any given frequency, their results show that placing more weight on social welfare would decrease both airline and HSR fares. A more recent effort is in Adler et al. (2014), who propose a differentiated Bertrand network model capable of not only dealing with the inter-modal competition but also investigating the impacts of international air transport liberalization, regional open skies policies, and airport slot allocation in East Asia. It is found that HSR profit will be substantially reduced with bi-lateral air liberalization, a single aviation market, or the introduction of Maglev services.

In a more general setting, Jørgensen and Mathisen (2013) address the competition between two transport firms each maximizing a goal function defined as weighted sum of profit, revenue, and total consumer surplus. However, the goal and cost functions are assumed to be identical across operators. The competition is modeled under three situations: Cournot, Bertrand, and Collusion. The results endorse collusion between firms if they produce complementary goods. Socorro and Viecens (2013) compare integration and competition of HSR and air transport. While travelers’ decisions are based on generalized costs, Jørgensen and Mathisen (2013) and Socorro and Viecens (2013) consider demand functions only sensitive to fares. Furthermore, these studies do not consider competition on service frequency. This is certainly less realistic, because transportation operators usually first determine their service frequency at a more strategic level, and then set price which is more flexible to change.

We find some important gaps that remain despite the large body of literature on air-rail competition. First, modeling the sequential determination of multiple service attributes is more realistic than simultaneous or partial consideration of those attributes, the latter however dominating the literature. The choice of train technologies, which is of special interest to strategic rail planning, is neglected. Second, how the ownership nature of the rail agency, as reflected by its operating objective, shapes the service attributes of rail and its competitiveness in intercity travel needs further investigation. Third, the importance of rail station access to the attractiveness of rail service is not well understood. Our study attempts to fill these gaps.

### 3 The model

To account for the sequential decision process of the rail agency and competing airline, we develop a three-stage game-theoretic model, in which the rail agency first chooses its optimal train technology (and thus speed) at the top stage. Given the train technology, the middle stage pertains to setting service frequency by both the rail agency and the competing airline. At the bottom stage, air and rail fares are determined. We adopt backward induction to solve the three-stage game: the fare determination problem is first solved for a perfect Nash equilibrium, assuming a given train technology and air and rail service frequencies. The equilibrium fares are then employed to find equilibrium service frequencies. Because only a limited number of train technologies (i.e., conventional rail, HrSR, HSR) will be considered, the top-stage decision making is performed by enumerating these train technologies. In other words, we repeatedly solve the two-stage game on frequency and fare for each train technology. The technology that yields the highest value for the rail agency will be finally chosen.

To set up the gaming framework, we first specify demand and cost functions and the objectives of the air and rail operators. Then, we derive the equilibrium fare for the bottom stage, and analyze the solutions. Deriving close-form expressions for equilibrium frequency is impossible due to highly non-linear functional
forms and a multitude of parameters involved. Consequently, we resort to numeric analysis, which is relegated to Section 4. Finally, the objectives of the rail agency and airline are specified at the top stage.

3.1 Setup

As mentioned before, we consider an intercity travel market with one airline and one rail agency providing transportation services. The presence of a single airline best represents corridors that connect between a city which is an airline’s hub and a spoke city. Following Singh and Vives (1984), we assume a continuum of identical consumers within the market. The generalized travel cost to a representative consumer consists of out-of-pocket money and time-related costs involved in travel, for both rail and air:

\[
\bar{p}_i = p_i + \mu_1 w_i + \mu_2 \frac{l_i}{s_i} + \mu_3 \frac{T_i}{4f_i}, \quad i = r, a
\]  

(1)

where \(\bar{p}_i \ (i = r \text{ (rail), } a \text{ (air)})\) is the generalized travel cost for the representative customer taking mode \(i\); \(p_i\) the fare of mode \(i\); \(w_i\) the sum of rail station/airport access and egress time of mode \(i\); \(s_i\) the speed of mode \(i\); \(l_i\) travel distance of mode \(i\). Therefore, \(\frac{l_i}{s_i}\) describes line-haul trip time of mode \(i\). The last component in (1) describes passenger schedule delay, defined as the difference between a traveler’s desired departure time and the closest scheduled departure time. Although individual travelers care about their specific departure time, the inverse of service frequency \(f_i\) is widely used to capture the aggregate average schedule delay (e.g., Abrahams, 1983; Brueckner and Flores-Fillol, 2007; Brueckner and Girvin, 2008; Eriksen, 1978; Richard, 2003), which is intuitive if one considers uniform flight/train departures and passenger demand along a time circle of length \(T_i\) (which is the total operating hours of a day). Then, the average schedule delay per traveler equals \(\frac{T_i}{4f_i}\).

\(\mu_i \ (i = 1, 2, 3)\) denotes unit time values of travelers for rail station/airport access and egress, line-haul travel time, and schedule delay respectively.

A representative consumer makes travel decisions by maximizing his/her utility, which is assumed to have the following quadratic and strictly concave functional form, subject to budget constraint:

\[
\max_{q_0,q_r,q_a} U = q_0 + \alpha_a q_a + \alpha_r q_r - \frac{b_a q_a^2 + 2c a q_a r + b_r q_r^2}{2}
\]

\[
\text{s.t. } \quad q_0 + \bar{p}_a q_a + \bar{p}_r q_r = I
\]  

(2)

where \(q_0\) is the consumption of numeraire good; \(q_a\) and \(q_r\) the consumption of air and rail services; \(\alpha_a, \alpha_r, b_a, b_r, c\) and \(I\) positive parameters; and \(I\) the total time and monetary budget of the representative consumer. Concavity of the utility function requires that \(b_a b_r - c^2 > 0\). In this study, we consider a stronger case: \(b_a > c\) and \(b_r > c\). Taking first-order conditions of (2) yields the functions of inverse demand for air and rail:

\[
\bar{p}_a = a_a - b_a q_a - c q_r
\]  

(3.1)

\[
\bar{p}_r = a_r - b_r q_r - c q_a
\]  

(3.2)

The second order conditions of (2) are guaranteed since the Hessian is negative definite given the concavity condition of the utility function. After some simple algebra, we obtain the following demand functions:

\[
q_r = G_r(\bar{p}_r, \bar{p}_a) = \alpha_0 - \alpha_1 \bar{p}_r + \tau \bar{p}_a
\]  

(4.1)

\[
q_a = G_a(\bar{p}_r, \bar{p}_a) = \beta_0 - \beta_1 \bar{p}_a + \tau \bar{p}_r
\]  

(4.2)

where \(\alpha_0 = \frac{(a_a b_r - a_r c)}{m}, \ \beta_0 = \frac{(a_a b_r - a_a c)}{m}, \ \alpha_1 = \frac{b_a}{m}, \ \beta_1 = \frac{b_r}{m},\) and \(\tau = \frac{c}{m}\), with \(m = b_a b_r - c^2\). Because \(b_a > c\) and \(b_r > c\), \(\alpha_1 > \tau\) and \(\alpha_2 > \tau\). And air and rail services are imperfect substitutes. We assume \(a_r b_a - a_a c > 0\) and \(a_a b_r - a_r c > 0\) so that the intercepts of both demand functions are positive. With this demand specification, total market-level demand varies for different values of \(\bar{p}_r\) and \(\bar{p}_a\).
On the supply side, we suppose that the operating cost of a flight or train trip is given by \( C_i = K_i + c_i n_i \) (\( i = r, a \)), where \( K_i \) is the fixed cost independent of the flight/train size; \( c_i \) the marginal cost per seat; \( n_i \) the average size of the flight/train, measured in the number of seats onboard. Under the assumption of load factor equal to 1, and air demand \( q_i \) (\( i = r, a \)) fulfills \( f_i n_i \). This assumption can be relaxed if we instead consider a predetermined target load factor less than one, as is often the case for planning purposes. However, this relaxation will not change the insights about air-rail competition. The fixed cost includes elements such as depreciation (GRA Inc., 2004). For the rail agency, \( K_r \) further includes track maintenance and renewal costs per train trip. The above cost function specification results in the following profit functions:

\[
\pi_i = (p_i - c_i)q_i - f_i K_i \quad i = r, a
\]  

(5)

While the airline is a pure profit maximizer, the objective of the rail agency is a weighted sum of profit \( \pi_r \) and consumer surplus \( CS_r \):

\[
\theta_r = (1 - \theta)\pi_r + \theta CS_r
\]  

(6)

where \( \theta \) is the weighting parameter. Obviously, if the rail agency is a pure profit-driven entity, then \( \theta \) equals 0. If the rail agency is a public entity aiming at maximizing social welfare, then \( \theta = 0.5 \). If the objective of the rail agency is a mix of maximizing profit and social welfare, then the value of \( \theta \) will be somewhere between 0 and 0.5.

To express consumer surplus for rail users, we need to first calculate the intercept of rail demand function (7).

\[
q_r = 0 \rightarrow \bar{p}_r^0 = \frac{a_0}{a_1} + \frac{\tau p_a}{a_1} = \frac{a_0}{a_1} + \frac{\tau}{a_1} \left( p_a + \mu_1 w_a + \mu_2 \frac{I_a}{s_a} + \mu_3 \frac{\gamma_a}{4f_a} \right)
\]  

(7)

Consequently, consumer surplus for rail passengers could be derived as:

\[
CS_r = \int_{\rho_r^0}^{\rho_r^\infty} G_r(z, \bar{p}_a)dz = \int_{\rho_r^0}^{\rho_r^\infty} (a_0 - \alpha_1 z + \tau \bar{p}_a)dz = \tau \bar{p}_a (\bar{p}_r^0 - \bar{p}_r) - \frac{\tau}{2} \tau \bar{p}_r \left( \bar{p}_r^0 - \bar{p}_r \right)^2 + a_0 (\bar{p}_r^0 - \bar{p}_r)
\]  

\[
= \frac{1}{\alpha_1 \frac{\beta_1}{\beta_2} \frac{f_r}{f_a}} \left( f_r \frac{s_r}{\alpha_1 \left( 4 \mu_3 s_a T_a + 4 f_a (p_a s_a + \mu_1 s_a w_a + I_a) \right) - a_1 f_a \frac{s_a}{\alpha_1} (\mu_3 s_r T_r + 4 f_r (p_r s_r + \mu_4 s_r w_r)) \right)^2
\]  

(8)

### 3.2 Fare competition

The bottom stage of the game pertains to determining rail and air fares, assuming that train technology and rail and air service frequencies are given. The first order conditions for (5) and (6) yield the best fare response functions, as shown in (9.1) and (9.2).

\[
p_r = \frac{1}{4a_1 f_a s_a} \left( f_r s_r (2\theta - 1) \left( 4 \beta_1 f_a s_a + \tau \left( \mu_3 s_a T_a + 4 f_a (p_a s_a + \mu_1 s_a w_a)) \right) + \alpha_1 f_a s_a (\mu_3 s_r T_r (1 - 2\theta) + 4 f_r (p_r s_r + \mu_4 s_r w_r)) \right) \right)
\]  

\[
(9.1)
\]

\[
p_a = \frac{1}{8f_a s_a} \left( f_r s_r (2\theta - 1) \left( 4 \beta_1 f_a s_a + \tau \left( \mu_3 s_a T_a + 4 f_a (p_a s_a + \mu_1 s_a w_a)) \right) + \alpha_1 f_a s_a (\mu_3 s_r T_r (1 - 2\theta) + 4 f_r (p_r s_r + \mu_4 s_r w_r)) \right) \right)
\]  

\[
(9.2)
\]

To derive expressions for equilibrium rail and air fares, we simultaneously solve (9.1) and (9.2). This results in equilibrium fares \( p_r^* \) and \( p_a^* \) as presented in (10.1) and (10.2). In essence, the derived prices are the Nash equilibrium solution of a non-cooperative competition game.
Thus, the competing airline and air fares will be determined after service frequencies, it is interesting to understand how equilibrium fares change in response to frequency variations. To this end, we take the first order derivatives of (10.1) and (10.2) with respect to rail and air frequencies, and find the following:

**Proposition 1:** If the airline increases its service frequency, the equilibrium air fare will increase. The rail agency will lower its fare. The same is true for the rail agency: if rail service becomes more frequent, equilibrium rail fare will increase. In this case, the competing airline will reduce its fare.

**Proof:** Partial differentials of the equilibrium fares with respect to service frequencies are shown in (12.1)-(12.4). Note that when $\theta \in [0,0.5]$, $-(3\theta - 2) > (1 - 2\theta) \geq 0$. In addition, $2\alpha_1 \beta_1 > \tau^2 > 0$ (because $b_0 b_r < c^2 > 0$ as assumed by the concavity of the consumer utility function). Thus, the denominators in (12.1)-(12.4) are always negative. The signs of the numerators are found in a similar way. Consequently the sign for each partial differential is determined.

\[
\begin{align*}
\frac{\partial p^*_a}{\partial l_a} &= \frac{2\alpha_1 \beta_1 f_a s_a}{\tau (1-2\theta)+2\alpha_1 \beta_1 (3\theta-2)} \left( \frac{\mu T_r}{4f_a} \right) > 0 \quad (12.1) \\
\frac{\partial p^*_a}{\partial f_a} &= \frac{2\alpha_1 \beta_1 (1-2\theta)}{\tau (1-2\theta)+2\alpha_1 \beta_1 (3\theta-2)} < 0 \quad (12.2) \\
\frac{\partial p^*_r}{\partial l_r} &= \frac{2(\alpha_1 \beta_1 - \tau)(2\theta - 1)}{\tau (1-2\theta)+2\alpha_1 \beta_1 (3\theta-2)} \left( \frac{\mu T_r}{4f_a} \right) > 0 \quad (12.3)
\end{align*}
\]
The intuition behind (12.1)-(12.4) is as follows: once the service frequency for air is increased, the average schedule delay for air travelers will decrease, meaning improved quality of air service. Consequently air passengers will be willing to pay more (i.e., fare increase). To maintain competitiveness, the rail agency will lower its fare. The same applies when rail increases its service frequency.

As train technology (which determines train operating speed) and rail station/airport access are two focuses of this study, we further examine how the equilibrium fares respond to their changes. Intuitively, greater train operating speed, which shortens line-haul rail travel time, enhances the competitive position of rail. As a result, the rail agency can charge a higher price; in contrast, the air carrier will respond by reducing its fare. Reducing rail station/airport access time will increase the attractiveness of rail/air travel; thus the rail agency/airline can charge passengers high fare. We formalize these speculations as Propositions 2 and 3 below:

**Proposition 2:** Increasing train speed allows the rail agency to charge passengers higher fare. Meanwhile, airlines will reduce air fare.

**Proof:** We take partial differentials of the equilibrium fares with respect to train speed, as shown in (13.1)-(13.2). Signs are determined in a similar way as in Proposition 1. ■

\[
\frac{\partial p_a}{\partial s_r} = \frac{a_1 \tau (1-\theta)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \frac{\mu_r}{s_r} \right) < 0
\]  
(13.1)

\[
\frac{\partial p_r}{\partial s_r} = \frac{(2 \alpha_1 \beta_1 - \tau^2) (2 \theta - 1)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \frac{\mu_r}{s_r} \right) \geq 0
\]  
(13.2)

**Proposition 3:** Decreasing airport access time would allow the airline to charge higher fare. In response, the rail agency will decrease rail fare. The reverse is true when train station access time is decreased.

**Proof:** Again, we take partial differentials of equilibrium fares with respect to access time to the airport (train station). The results are shown in (14.1)-(14.4). Signs are determined in a similar way as in Proposition 1. ■

\[
\frac{\partial p_a}{\partial \omega_a} = \frac{\tau^2 (2 \theta - 1) + a_1 \beta_1 (2 - 3 \theta)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \mu_1 \right) < 0
\]  
(14.1)

\[
\frac{\partial p_r}{\partial \omega_a} = \frac{\tau \beta_1 (2 \theta - 1)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \mu_1 \right) \geq 0
\]  
(14.2)

\[
\frac{\partial p_a}{\partial \omega_r} = \frac{(2 \alpha_1 \beta_1 - \tau^2) (1 - 2 \theta)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \mu_1 \right) \leq 0
\]  
(14.3)

\[
\frac{\partial p_r}{\partial \omega_r} = \frac{a_1 \tau (\theta - 1)}{\tau^2 (1-2 \theta) + 2 \alpha_1 \beta_1 (3 \theta - 2)} \left( \mu_1 \right) > 0
\]  
(14.4)

### 3.3 Frequency competition

The middle stage of the three-stage game pertains to competition on service frequency. Using backward induction, we plug the equilibrium fares, i.e., (8.1) and (8.2), into air and rail objective functions, which now become functions of only frequencies, for a given train technology. As frequencies are finite, we assume that daily service frequencies for both air and rail have bounds \((f_a^{\min}, f_a^{\max}, f_r^{\min}, f_r^{max})\). The game at this stage can be presented by (15.1)-(15.4).

**Air:**

\[
\max_{f_a} \pi_a (f_a, f_r)
\]

**s. t.** \(f_a^{\min} \leq f_a \leq f_a^{max}\)

**Rail:**

\[
\max_{f_r} \pi_r (f_a, f_r)
\]

\[
\max_{f_r} \pi_r (f_a, f_r)
\]

\[
\max_{f_a} \pi_a (f_a, f_r)
\]

\[
\max_{f_a} \pi_a (f_a, f_r)
\]
\[ s.t. \quad f_r^{\min} \leq f_r \leq f_r^{\max} \quad (15.4) \]

In the above frequency game, the strategy sets for the airline and the rail agency are bounded, closed, and convex. In addition, the payoff functions for the rail agency and the airline are continuous. Therefore, at least one Nash equilibrium (in the form of either a pure or mixed strategy) exists (Glicksberg, 1952). Focusing on pure strategies, the necessary conditions for the optimization problem facing the airline and the rail agency can be described by the Karush-Kuhn-Tucker (KKT) conditions, following Hillier and Lieberman (2001). They are shown in (16.1)-(16.13), where \( \lambda^i \)'s \((i = a, r; j = 1,2)\) are the Lagrangian multipliers.

\[
\begin{align*}
\frac{\partial \pi_a}{\partial f_a} + \lambda^1_2 - \lambda^2_2 & \leq 0 \quad (16.1) \\
\frac{\partial \pi_a}{\partial f_a} + \lambda^1_a - \lambda^2_a & = 0 \quad (16.2) \\
f_a & \geq f_a^{\min} \quad (16.3) \\
\lambda^1_a (-f_a + f_a^{\min}) & = 0 \quad (16.4) \\
f_a & \leq f_a^{\max} \quad (16.5) \\
\lambda^1_a (f_a - f_a^{\max}) & = 0 \quad (16.6) \\
\frac{\partial \pi_r}{\partial f_r} + \lambda^1_r - \lambda^2_r & \leq 0 \quad (16.7) \\
f_r (\frac{\partial \pi_r}{\partial f_r} + \lambda^1_r - \lambda^2_r) & \leq 0 \quad (16.8) \\
f_r & \geq f_r^{\min} \quad (16.9) \\
\lambda^1_r (-f_r + f_r^{\min}) & = 0 \quad (16.10) \\
f_r & \leq f_r^{\max} \quad (16.11) \\
\lambda^1_r (f_r - f_r^{\max}) & = 0 \quad (16.12) \\
\lambda^1_i & \geq 0 \quad i = a, r \text{ and } j = 1,2 \quad (16.13)
\end{align*}
\]

In this study, we use a trial-and-error approach (Hillier and Lieberman, 2001) to solve for \( f_a \) and \( f_r \) from the KKT conditions. Note that, under equilibrium, at most one of the rail frequency constraints (16.3) and (16.5) is binding; similarly, at most one of the air frequency constraints (16.9) and (16.11) is binding. Consequently, we can have nine possible combinations for \( f_a \) and \( f_r \): \{\( f_a = f_a^{\min}; f_a^{\min} < f_a < f_a^{\max}; f_a = f_a^{\max} \)\} coupled with \{\( f_r = f_r^{\min}; f_r^{\min} < f_r < f_r^{\max}; f_r = f_r^{\max} \)\}. For each combination, we solve for \( \lambda^i \)'s \((i = a, r; j = 1,2)\) without imposing non-negativity constraints for the \( \lambda^i \)'s. If the obtained \( \lambda^i \) is negative, then that combination is infeasible. Otherwise, we further check the second-order conditions of the rail and air objective functions for local optimality.

An alternative approach to solve for equilibrium frequencies is to explicitly consider frequencies as integers. Then the frequency competition can be modeled as a matrix game, in which the airline and the rail agency each has a finite set of strategies. This game is represented as below:

\[
\begin{align*}
S_a &= \{f_a^{\min}, \ldots, f_a^{\max}\} \quad (17.1) \\
S_r &= \{f_r^{\min}, \ldots, f_r^{\max}\} \quad (17.2) \\
Br_a(f_r) &= \arg\max_{f_a \in S_a} \pi_a(f_a, f_r) \quad (17.3) \\
Br_r(f_a) &= \arg\max_{f_r \in S_r} \pi_r(f_a, f_r) \quad (17.4)
\end{align*}
\]

where \( S_i \((i = a, r)\) is the strategy set for mode \( i \); \( Br_i(f) \) is the best frequency response of the airline given rail's frequency; likewise for \( Br_i(f_a) \). A strategy pair \{\( f_a^*, f_r^* \)\} gives a pure strategy Nash Equilibrium if \( f_a^* \in Br(f_r^*) \) and \( f_r^* \in Br(f_a^*) \). To solve the matrix game, we first incorporate equilibrium price expressions (8.1) and (8.2) into rail and air payoff functions. Each payoff then becomes only a function of rail and air frequencies (for a
given train technology}. We vary the rail frequency between \( f_r^{min} \) and \( f_r^{max} \). For a given rail frequency, we identify the best air frequency response. The best frequency response for rail is determined similarly. A pair of frequencies each being the best frequency response to the other mode’s frequency then presents an equilibrium frequency solution.

### 3.4 Train technology determination

As mentioned before, there is limited room for changing aircraft speed. The top stage is simply determining which train technology (speed) would result in the best objective value for the rail agency, anticipating the equilibrium frequency and fare responses in the air-rail competition. The objective is shown in (18):

\[
\max_{\theta_r} \theta_r = (1 - \theta)\mu_r + \theta CS_r 
\]  

Recall that only three train technologies are considered: conventional rail, HrSR, and HSR. Instead of performing explicit optimization of (18), we solve for the frequency and fare competition games under each train technology. The chosen technology is the one that gives the greatest value for \( \theta_r \).

### 4 Numerical analysis

This section demonstrates applications of the three-stage air-rail competition model. We first describe the types of markets to be investigated and values for the model parameters. We then present the equilibrium outcomes and discuss the results. The impact of rail station access on market share and rail/air objective values is further assessed.

#### 4.1 Model setup

We consider four hypothetical corridors with distance ranging from 284 to 500 miles, which are comparable to four actual corridors connecting Chicago and four major cities in the US Midwest region (Table 1). Currently, these corridors are served by both air and conventional Amtrak service. We further analyze two longer corridors in the Appendix. Upgrading conventional rail to HrSR with a maximum speed of 110 mph on these corridors is under consideration (TEMS, 2004). In what follows, we investigate cases that conventional rail, HrSR, and HSR can all operate on these corridors. Rail travel times for conventional rail and HrSR are directly borrowed from TEMS (2004). TranSystems (2009) provides HSR travel time for Chicago-St Louis corridor, based on which HSR travel times for other corridors are estimated proportional to distance. Information on air distance is obtained from the US Bureau of Transportation Statistics website (BTS, 2015).

Scheduled flight time is calculated using the following formula (Zou, 2012):

\[
\text{Scheduled flight time (min)} = 39.90 + 0.12 \times \text{Distance (miles)} \quad (17)
\]

#### Table 1: Rail and air distance and travel time in corridors considered

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Comparable corridors in the US Midwest</th>
<th>Rail transport</th>
<th>Air transport</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (miles)</td>
<td>Conventional</td>
<td>HrSR</td>
</tr>
<tr>
<td>1 Chicago-St. Louis</td>
<td>284</td>
<td>5:30</td>
<td>3:50</td>
</tr>
<tr>
<td>2 Chicago-Cleveland</td>
<td>341</td>
<td>7:15</td>
<td>4:25</td>
</tr>
<tr>
<td>3 Chicago-Minneapolis</td>
<td>443</td>
<td>8:15</td>
<td>5:40</td>
</tr>
<tr>
<td>4 Chicago-Omaha</td>
<td>500</td>
<td>8:35</td>
<td>7:05</td>
</tr>
</tbody>
</table>

The parameters related to time and time values in the traveler generalized cost function (1) are determined based on existing estimates in the literature (Table 2). More specifically, the three time value parameters \((\mu_1, \mu_2, \mu_3)\) draw from Cascetta and Coppola (2012) and US DOT (2014). Rail station and airport access time and service hours in a day \((w_r, w_a, T_r, T_a)\) follow those in Yang and Zhang (2012).
Table 2: Parameter values for the numeric example

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>Access time value</td>
<td>$56.7/\text{hr}$</td>
<td>Cassetta and Coppola (2012); US DOT (2014)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Flight/line-haul time value</td>
<td>$49.3/\text{hr}$</td>
<td>US DOT (2014)</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>Schedule delay time value</td>
<td>$48.7/\text{hr}$</td>
<td>Cassetta and Coppola (2012); US DOT (2014)</td>
</tr>
<tr>
<td>$w_r$</td>
<td>Rail access time</td>
<td>1 hr</td>
<td>Yang and Zhang (2012)</td>
</tr>
<tr>
<td>$w_a$</td>
<td>Air access time</td>
<td>3 hr</td>
<td>Yang and Zhang (2012)</td>
</tr>
<tr>
<td>$T_e = T_a$</td>
<td>Operating hours of a day</td>
<td>16 hr</td>
<td>Yang and Zhang (2012)</td>
</tr>
</tbody>
</table>

Following Zou and Hansen (2012), we assume the fixed flight cost to be $1250/\text{hr}$, and the variable cost to be $38/\text{seat-hr}$ (updated to 2015 value with 3% inflation rate). Based on these values and the scheduled flight time, we estimate $K_d$ and $c_a$ for a flight to service each corridor (note that $K_a$ and $c_a$ are in $/\text{seat}$). As mentioned before, fixed cost for each train trip ($K_e$) includes depreciation cost and track maintenance and renewal cost. For depreciation cost, we first amortize the train purchase price (IDOT, 2012; UIUC and UIC, 2013; US DOT, 2011; TEAMS, 2010) over a life cycle of 30 years with 5% interest rate. In view of the rail travel time in Table 1, we assume that each conventional and HrSR train can run one round trip per day. In contrast, a HSR train can make two daily round trips. Depreciation cost components for conventional, HrSR, and HSR are $2583$ (air), $2583$ (air) and $1607$ per train pair, respectively. To estimate track maintenance and renewal cost for conventional and HrSR trains, we follow Zambreski (2004). Assuming concrete ties, dominant freight traffic, and moderate curves, we compute the unit costs for conventional rail and HrSR as $2.45$ and $4.07$ per train.mile. In terms of track maintenance and renewal cost for HSR, we inflate the value in Levinson et al. (1997) to 2015 and use $2583/\text{train.mile}$. For each corridor and train technology, we then multiply the corresponding unit cost by the corridor length and find corridor-specific track maintenance cost. Adding depreciation cost to track maintenance cost, we find the value of $K_e$ for each train type and corridor, as presented in Table 3. The variable cost ($c_r$) is calculated using cost factors from TEMS (2010) and the distance for each corridor.2

We determine the values for $\alpha_i$, $\beta_i$ ($i = 0, 1$), and $\tau$ such that the generated equilibrium riderships for air and rail are similar to the actual riderships in comparable corridors in Table 1. The actual ridership data are obtained from TEMS (2004). Table 3 documents the parameter values for $\alpha_i$, $\beta_i$, and $\tau$, along with values for $K_d$, $c_a$, and $c_r$.

Table 3: Parameter values used in numerical analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>$\alpha_1$</th>
<th>$\beta_1$</th>
<th>$\tau$</th>
<th>$K_d$/\text{seat}</th>
<th>$K_r$/\text{HV}</th>
<th>$c_r$/\text{HV}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3575</td>
<td>3700</td>
<td>13.5</td>
<td>6.75</td>
<td>1500</td>
<td>45</td>
<td>3278</td>
<td>3738</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3191</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>2950</td>
<td>2750</td>
<td>9</td>
<td>4.5</td>
<td>1650</td>
<td>49</td>
<td>3418</td>
<td>3970</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3509</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>6200</td>
<td>5500</td>
<td>17</td>
<td>8.5</td>
<td>1700</td>
<td>51</td>
<td>3668</td>
<td>4386</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4079</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>2375</td>
<td>2250</td>
<td>6</td>
<td>3</td>
<td>1900</td>
<td>57</td>
<td>3808</td>
<td>4618</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4397</td>
<td>65</td>
</tr>
</tbody>
</table>

4.2 Equilibrium outcomes

We use the above parameter values and solve for the three-stage model. At the frequency competition stage, we follow the discussions in subsection 3.3 and consider both continuous and discrete frequency values. The lower and upper bounds for air ($f_a^{\min}$ and $f_a^{\max}$) are considered to be 1 and 30 per day; the same for rail. When continuous frequencies are considered, the second-order derivative of the rail (air) objective with respect to rail (air) frequency is always negative within the feasible frequency range, which suggests that the frequencies obtained from solving the KKT conditions will be Nash equilibrium frequencies. When frequencies are restricted to integer values, the resulting equilibrium frequencies are very similar to those in the continuous

---

2 $c_r$ covers cost related to train equipment maintenance, train engine crew, fuel, on-board services, and insurance, each expressed in $/\text{seat}$. Insurance is assumed the same for all train types. The cost of on-board services is the same for conventional rail and HrSR, but lower for HSR. HrSR has the greatest fuel cost. However, train engine crew cost and equipment maintenance cost decrease with speed (TEMS, 2010). Aggregating these cost components, the net result is that $c_r$ ($/\text{seat}$) decreases from conventional rail to HrSR to HSR (Table 2). The decreasing trend of $c_r$ with train speed is also consistent with findings in the literature (e.g., see Civity (2013)).
The discrete frequencies under equilibrium are found as the rounding results of the continuous equilibrium frequencies.

In what follows we present the equilibrium outcomes including objective function values, frequencies, fare, and market share for air and rail, for all the four scenarios. Since the impact on the equilibrium outcome of the rail agency objective is one focus of our study, every result is presented under $\theta = 0$ and 0.5, which correspond to the rail agency being a pure profit maximizer and a pure social welfare maximizer. We have also experimented with other $\theta$ values between 0 and 0.5, which mean that the rail agency has mixed private and social goals. Not surprisingly, the results lie between those with $\theta = 0$ and 0.5. For brevity results with $\theta = 0$ and 0.5 are reported.

Let us first look at the objective function values for the rail agency and the airline, which are shown in Table 4. Several points are worth noticing. First, for an equilibrium to be meaningful, any private operator must realize positive payoff. We find no meaningful equilibrium for competition between private conventional rail and air transport. Other equilibrium outcomes are meaningful. Conditional on availability of high speed infrastructure, the train technology that yields the highest objective value is always HSR, regardless of whether the agency is a profit or social welfare maximizer. The rail objective value increases with train speed; whereas the airline profit shows a reverse relationship. This is not surprising: faster train service reduces line-haul travel time and thus the generalized travel cost of rail passengers, which makes rail more attractive. As a consequence, some travelers divert from air to rail; new demand is induced (this is shown explicitly in Figure 4). The change in demand leads to increased revenue for the rail agency and consumer surplus of rail passengers but significant profit drop for the airline.

Second, for a given train technology, the equilibrium value for the rail objective does not vary significantly between the rail agency being a pure profit maximizer and a pure social welfare maximizer. This may be attributed to two opposing effects. Suppose we convert the rail agency from a profit maximizer ($\theta = 0$) to a social welfare maximizer ($\theta = 0.5$). On the one hand, after conversion the rail agency will put more focus on attracting travelers by lowering fare and increasing frequency, leading to reduced profit; on the other hand, this results in greater rail ridership and thus consumer surplus, which contributes to increased rail objective value. The two effects are corroborated by the equilibrium frequency, fare, and market share under the two $\theta$ values (Figures 1–3). After conversion the rail agency always realizes negative profit which calls for government financial assistance to be provided. We also note that airline profit would substantially drop after the conversion, due to more intense fare and frequency competition which tends to lower revenue and increase operating cost of the airline.

Third, with conventional rail and HrSR the rail objective value is much lower than the airline profit, which is consistent with rail’s smaller market share. This is particularly eminent for the longer corridors, i.e., scenarios 3 and 4. If HSR is deployed, the variable rail cost becomes much lower than that of the airline. This further improves the competitiveness of rail and results in greater objective values for rail.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\theta$</th>
<th>Conventional rail</th>
<th>HrSR</th>
<th>HSR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\pi_a$</td>
<td>$O_r$</td>
<td>$\pi_r$</td>
</tr>
<tr>
<td>S1</td>
<td>0</td>
<td>0</td>
<td>93.6</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>No equilibrium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
<td>0</td>
<td>131.8</td>
<td>-4.0</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>No equilibrium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>0</td>
<td>0</td>
<td>371.9</td>
<td>-5.4</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>No equilibrium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>0</td>
<td>0</td>
<td>171.0</td>
<td>-3.7</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>No equilibrium</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 presents the equilibrium market shares. As discussed above, the conventional rail $\rightarrow$ HrSR $\rightarrow$ HSR conversion makes rail more attractive due to reduced travel time and operating cost, leading to greater rail market share. When $\theta = 0.5$, the extent of market share increase is generally over 100% from conventional rail to HSR (as indicated by the length of the corresponding red bars). In fact, as consumer surplus is part of the overall social welfare, maximizing social welfare ($\theta = 0.5$) puts more emphasis on attracting passengers,
resulting in greater rail market share than profit maximization ($\theta = 0.5$). Indeed, rail would become the dominant mode when $\theta = 0.5$ with HSR technology under all scenarios.

When $\theta = 0$, the equilibrium frequencies of rail and air are generally comparable for scenarios 1, 2, and 4, ranging from 6 to 11 per day. In scenario 3, where actual ridership (under Conventional rail, is much greater than other scenarios, air frequency is about 1.5 times of rail frequency. If the rail agency cares equally about consumer surplus and profit, i.e., $\theta = 0.5$, rail frequency will increase. The competition pressure also prompts the airline to adjust frequency. In our case, the response for air is to reduce flight frequency, but to a less extent than the increase in rail frequency. Average aircraft size, which is equal to the number of air passenger divided by flight frequency, varies between 63 (scenario 1) and 137 (scenario 3). This is a plausible aircraft size range for the US market, where airlines mostly use short-range narrow-body jets (e.g., Airbus A319 and Boing 737) or regional jets (e.g., Bombardier CRJ700 and Embraer ERJ170) for stage lengths presented in Table 1.

Similar distribution patterns are observed in equilibrium fare. When $\theta = 0$, rail and air fares are very close to each other. However, if maximizing social welfare becomes the objective (i.e., $\theta = 0.5$), then rail fare will substantially decrease. This also puts pressure on the airline to cut price. Note that when $\theta = 0.5$, the equilibrium rail fare expression (10.1) collapses to $c_F$. In other words, the rail agency only charges each passenger the variable cost (or marginal cost) per seat. Whereas rail passengers are likely to benefit from lower fare along with greater service frequency, the revenue collected by the rail agency does not fully cover its
operating cost (see $\pi_r$ in Table 4). To make the rail agency break-even, government subsidy would be needed. Alternatively, the rail agency may adjust the weighting parameter $\theta$ to meet the self-finance requirement.

Figure 3: Equilibrium rail and air fare

Figure 4 shows that increasing the focus on consumer surplus, i.e., increasing the $\theta$ value, leads to more rail and system total demand, for scenario 1 under HSR. Similar trends are found for other scenarios. The left panel shows the percentage change in rail demand as compared to $\theta = 0$, which increases at a faster rate with larger $\theta$ values. The rail ridership increase comes from two sources: 1) modal shift from air; and 2) induced demand. We observe that if the rail objective shifts from profit maximization to social welfare maximization, rail ridership will grow by 92%. Overall system demand increase is also significant, by more than 40% between $\theta = 0$ and $\theta = 0.5$ as shown in the right panel.

Figure 4: Percentage demand increase with $\theta$ (baseline: $\theta = 0$) for scenario 1

Figure 5 illustrates direct and cross price elasticity values for scenario 1 under HSR. Other scenarios also offer similar insights. Inferred elasticity values are mostly in line with previous estimates (Brons et al., 2002; Fu et al., 2014; Gillen et al., 2004; Mumbower et al., 2014; Zou and Hansen, 2012). Air travel demand is always elastic with respect to airfare; whereas rail demand is elastic, with respect to rail fare, only when $\theta$ is less than 0.2. Increasing $\theta$ reduces rail travel cost which in turn elevates rail travel demand. It can be easily shown that $\frac{\partial q_i}{\partial p_i}$ ($\forall i = r, a$) is fixed (see (4.1) and (4.2)). Therefore, absolute value of rail price elasticity $\left| \frac{\partial q_r}{\partial p_r} \right|$ reduces with $\theta$. Similar reasoning can be made for air price elasticity. For any given $\theta$ value, air/rail demand is inelastic.
with respect to the price of the competing mode. As expected, absolute value of direct elasticity is always greater than the value of cross elasticity, regardless of \( \theta \) value and travel mode.

\[ \theta \]

**Figure 5: Direct and cross price elasticities of demand with \( \theta \) for scenario 1**

We plot direct and cross frequency elasticities of demand with \( \theta \) in Figure 6. We find that air/rail demand is always inelastic with respect to frequency, indicating that frequency cannot grow infinitely. Air frequency elasticity substantially increases with increasing \( \theta \) value; whereas increasing \( \theta \) reduces rail frequency elasticity. For a given mode and \( \theta \), the absolute value of cross elasticity is generally about half the corresponding direct elasticity value. The obtained frequency elasticity values for air are consistent with previous findings; whereas frequency elasticity for rail seems somewhat lower than previous estimates (Adler et al., 2010; Givoni and Rietveld, 2009; Pels et al., 2000; Pels et al., 2009; and Zou, 2012).

\[ \theta \]

**Figure 6: Direct and cross frequency elasticities of demand with \( \theta \) for scenario 1**

### 4.3 Impact of rail station access/egress time

One of the advantages of rail in intercity travel is shorter access time to rail stations as compared to airports, because rail stations are typically located in the city center. In fact, it has been argued that HSR would become unattractive to many travelers if the origin and/or destination station is outside a city’s downtown (Givoni and Banister, 2012). In Figure 7 we show how rail market shares will change by varying the rail access and egress time (i.e., \( w_c \)) to be 2 hours, 1 hour, and 30 minutes. By setting 1 hour as the baseline, Figures 8 and 9 further demonstrate the percentage change in air and rail objective function values when reducing rail access/egress time to 30 minutes and increasing it to 2 hours.
Because shorter rail access/egress time reduces generalized travel cost for rail passengers, we expect greater rail market share, as shown in Figure 7. Reducing rail station access time elevates rail market share for 3-4%; whereas increasing rail access time to 2 hours lowers rail market share by 6-9%. As shown in (14.3) and (14.4), the equilibrium rail and air fare will increase and decrease, respectively, after rail access/egress time is reduced. Our numerical analysis suggests similar responses of equilibrium frequency for rail and air. The fare and frequency responses (in absolute values) of air are always smaller than of rail. Comparing the two panels in Figure 7, the market share changes are not sensitive to whether the rail agency is a social welfare or profit maximizer.

\[ \theta = 0 \]

\[ \theta = 0.5 \]

Figure 7: Rail market share with different rail station access/egress times

Reducing rail access/egress time makes the rail agency better off, in spite of the fare increase. This is again regardless of whether the rail agency is a social welfare or profit maximizer (Figure 8). The reverse is true if the rail access/egress time is increased. When rail access/egress time is reduced from 1 hour to 30 minutes, the maximum percent increase in the rail objective value will be 27%; if the time increases from 1 hour to 2 hours, then the maximum percent reduction in the rail objective value will be 46%. Both occur to scenario 1.

Thus the elasticity of the rail objective to rail access/egress time for scenario 1 is about -0.46. Similar elasticities can be estimated for other scenarios and for the airline. We note that the extent of change in the rail objective value diminishes with distance, which is intuitive: the longer the corridor, the smaller portion of access/egress time in total travel time and generalized cost. The percentage change in airline profit is less significant when \( \theta = 0 \). The airline profit is more sensitive to rail access/egress time change with \( \theta = 0.5 \). This is probably due to greater sensitivity of air equilibrium fare to rail access/egress time when \( \theta = 0.5 \) (note that the sensitivity expression (14.4) reduces to \( \frac{\alpha e}{4 \alpha_x \beta_t - \frac{\tau^2}{\mu_1}}(\mu_1) \) when \( \theta = 0 \), which is smaller than \( \frac{\tau}{2 \mu_t}(\mu_t) \) with \( \theta = 0.5 \).
Figure 8: Percentage change in air and rail objective function values when rail station access/egress time decreased from 1 hour to 30 min

Figure 9: Percentage change in air and rail objective function values when rail station access/egress time increased from 1 hour to 2 hours

5 Conclusion

The continuous ridership growth in US intercity passenger rail transportation has spurred new development of HrSR and HSR lines, besides existing conventional rail services. However, it has been rarely scrutinized which train technology presents the best choice while rail competes with air services. In addition, the ownership of the rail agency that is in charge of the new rail service development can have non-trivial impact on the appeal of rail to travelers as well as the performance of the rail service. However, the impact is not well understood either. Furthermore, one advantage of rail over air travel is its shorter station access/egress distance and time, which contributes to the attractiveness of the rail mode. This study presents a stylized three-stage gaming model that explicitly investigates the above three aspects. We consider duopolistic intercity travel markets, where a rail agency competes with an airline. This best describes corridors that connect between a city which is an airline’s hub and a spoke city.

In the three-stage game, the rail agency first decides on the train technology. Given the technology, the rail agency and the airline compete sequentially on service frequency and fare. We find that HSR will always be the best train technology choice, in terms of rail market share and its overall objective value, conditioned by availability of high speed rail infrastructure. This holds whether the rail agency is private or public (or has a
mixed ownership). Having a public rail agency will greatly benefit rail travelers than having a private rail agency, in the form of reduced fare and more frequent services, which in turn result in greater demand and market share of rail. Making rail stations easier to access by reducing station access/egress time is found to substantially impact rail market share. The impact diminishes with the corridor distance.

These findings have practical implications for new intercity passenger rail development on corridors with presence of air services. However, some caveats should be exerted when further interpreting them. First, the conclusion that HSR is the best rail solution to compete with air is from the operational standpoint. The operational advantage of HSR over HrSR and conventional rail needs to be weighed against its usually greater costs associated with infrastructure investment (e.g., tracks and traffic control). Second, the considerable consumer benefits gained from running a public rail agency could mean reduced, or even negative, agency profit. In the latter case, government subsidy would take place. To justify government support for a new HSR service, the customer benefit gains should exceed the amount of subsidy, if there is any. If this condition is not met, then a compromise may be needed, for example, by making the rail agency "quasi-public", so that self-financing can be ensured. Third, if the rail service uses existing train stations that are located in the downtown, it often implies relatively good station access with provision of local transportation. If the rail station access is already significantly better than airport access, there may not be a strong need to further reduce rail station access/egress time, especially when it is costly to realize. The need can be further tempered for long corridors.

Acknowledgment

We thank two anonymous reviewers and especially Professor Chris Nash, the Associate Editor of the journal, for their constructive suggestions, which led to a substantial improvement of the paper.

Appendix

This appendix extends the range of corridors presented in Section 4. In particular, we consider two hypothetical corridors, with lengths of 650 and 800 miles, which are comparable to long corridors in Asia (e.g., Wuhan-Guangzhou (601 miles) and Beijing-Shanghai (809 miles) in China). Corridor lengths and travel times are presented in Table A.1. Cost parameters are presented in Table A.2. Other parameter values are the same as those in Table 2. $\alpha_0$, $\beta_0$, $\alpha_1$, $\beta_1$, and $\tau$ for the two scenarios are set such that the riderships under conventional rail become similar to riderships in scenario 3.

### Table A.1: Rail and air distance and travel time in corridors considered

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Length (miles)</th>
<th>Travel time (hr:min)</th>
<th>Rail transport</th>
<th>Air transport</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>HrSR</td>
<td>HSR</td>
</tr>
<tr>
<td>5</td>
<td>650</td>
<td>12:00</td>
<td>8:15</td>
<td>4:45</td>
</tr>
<tr>
<td>6</td>
<td>800</td>
<td>14:30</td>
<td>10:15</td>
<td>6:00</td>
</tr>
</tbody>
</table>

### Table A.2: Parameter values used in numerical analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>$\alpha_1 = \beta_1$</th>
<th>$\tau$</th>
<th>$K_a$ ($/seat$)</th>
<th>$K_C$ ($/seat$)</th>
<th>$c_C$ ($/seat$)</th>
<th>$c_C$ ($/seat$)</th>
<th>$c_C$ ($/seat$)</th>
<th>$c_C$ ($/seat$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6285</td>
<td>4750</td>
<td>16</td>
<td>6.5</td>
<td>2300</td>
<td>4175</td>
<td>5228</td>
<td>5234</td>
<td>69</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>6700</td>
<td>4425</td>
<td>10.5</td>
<td>5.25</td>
<td>2650</td>
<td>4543</td>
<td>5839</td>
<td>6071</td>
<td>85</td>
<td>76</td>
</tr>
</tbody>
</table>

Consistent with previous findings, we observe that HSR is the best choice for the rail agency, conditional on availability of high speed rail infrastructure. Figure A.1 shows rail and air market shares with $\theta$. Not surprisingly, we observe lower rail market shares for the longer-distance scenario, i.e., scenario 6. As the corridor length extends, the share of in-vehicle travel time cost in the passenger’s generalized cost will decrease. Due to shorter in-vehicle travel time of air, rail loses its relative advantage and rail market share...
drops. The difference between rail/air market shares in the two scenarios remains stable with increasing $\theta$ from 0 to 0.5.

Figure 5A.1: Rail and air market shares with $\theta$ for scenarios 5 and 6

Figures A.2 and A.3 show equilibrium fares and frequencies for scenarios 5 and 6. Compared to scenarios 1-4, both air and rail charge higher fares, which is intuitive because these corridors are longer and both air and rail operators realize greater cost parameters. The difference between rail frequencies in scenarios 5 and 6 insignificantly varies with $\theta$; whereas air frequencies of scenarios 5 and 6 converge with increasing $\theta$.

Figure 5A.2: Rail and air fares with $\theta$ for scenarios 5 and 6
References


IDOT, 2012. Chicago-St. Louis High-Speed Rail-Tier 1 Study. Illinois Department of transportation.